HALE SCHOOL

WA Exams Practice Paper E, 2015

Question/Answer Booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________

Your name ______________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 9 (6 marks)

(a) A and B are **acute** angles such that $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$.

(i) Determine the exact values of $\cos A$ and $\sin B$. (2 marks)

$$\cos A = \frac{4}{5}$$

$$\sin B = \frac{5}{13}$$

(ii) Show that the exact value of $\cos(A+B)$ is $\frac{33}{65}$. (2 marks)

$$\begin{aligned}\cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} \\ &= \frac{48-15}{65} \\ &= \frac{33}{65}\end{aligned}$$

(b) A and B are **obtuse** angles such that $\sin A = \frac{3}{5}$ and $\cos B = -\frac{12}{13}$. Determine the exact value of $\cos(A-B)$. (2 marks)

$$\begin{aligned}\cos(A-B) &= \cos A \cos B + \sin A \sin B \\ &= -\frac{4}{5} \times -\frac{12}{13} + \frac{3}{5} \times \frac{5}{13} \\ &= \frac{48+15}{65} \\ &= \frac{63}{65}\end{aligned}$$

Question 10 **(11 marks)**

(a) The graph of $y = a\,b^x$ passes through the points $P(2, -18)$ and $Q(3, -54)$.

i. Find the values of a and b. (3 marks)

$$y = -2\,(3)^x$$

ii. Determine the exact length of the line segment PQ. (2 marks)

$$PQ = \sqrt{(3-2)^2 + (-54 - -18)^2}$$

$$PQ = \sqrt{1297}$$

(b) Express $0.000\,023\,587$ in scientific notation to 3 significant figures. (2 marks)

$$2.35 \times 10^{-5}$$

(c) Describe the transformations required to change the graph of $y = 5^x$ into the graph of $y = 2 \times 5^{-x} - 9$. (4 marks)

Order:

Reflected in the y-axis.

Dilated vertically by s.f. 2

Translated down 9 units.

Question 11 **(9 marks)**

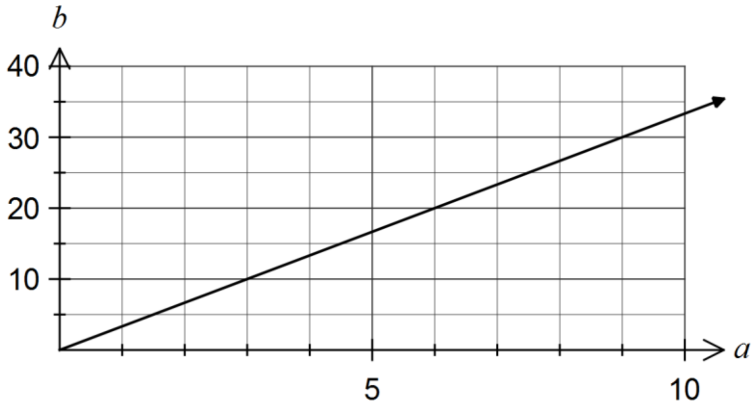
(a) The variable a is directly proportional to the variable b, such that when $a = 9$, $b = 30$.

(i) Determine an equation for the relationship between a and b. (2 marks)

$a = kb: \ 9 = k \times 30 \ \Rightarrow \ k = 0.3$

$a = 0.3b$

(ii) Sketch a graph of the relationship between a and b. (2 marks)



(b) The pressure, P, in an air bubble varies inversely with the volume, V, of the bubble. It is known that $P = 2.4$ kPa when $V = 5$ cm^3.

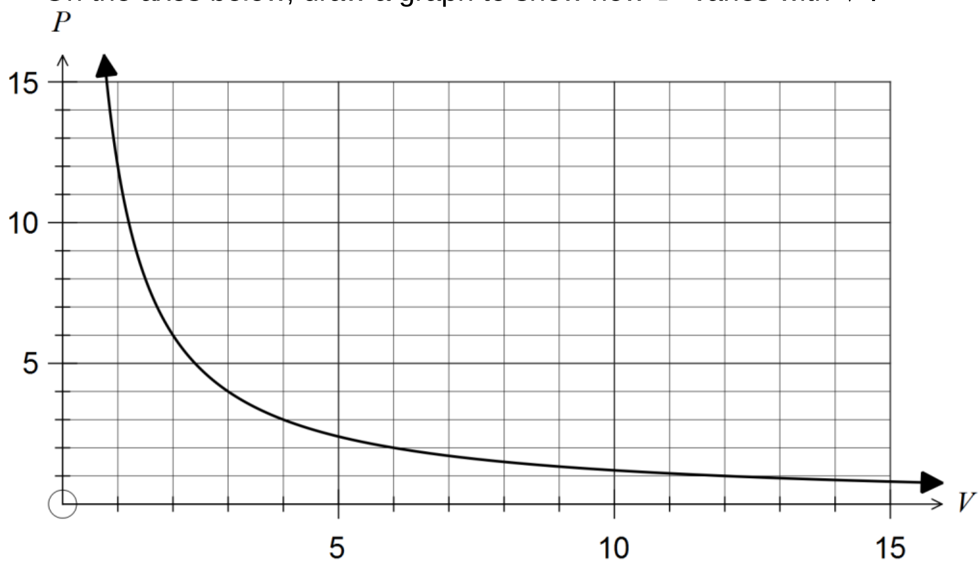
(i) Find the value of the constant k in the equation $P = \frac{k}{V}$. (1 mark)

$2.4 = \frac{k}{5} \ \Rightarrow \ k = 12$

(ii) Determine the value of V when $P = 10$ kPa. (1 mark)

$10 = \frac{12}{V} \ \Rightarrow \ V = 1.2$ cm^3

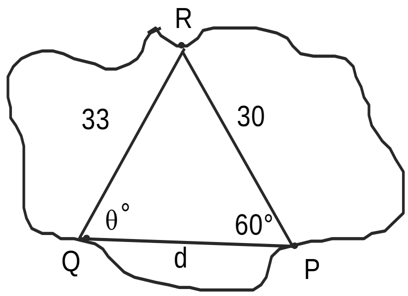
(iii) On the axes below, draw a graph to show how P varies with V. (3 marks)



Question 12 **(6 marks)**

P, Q and R are three campsites on the shore of a lake. The distance across the lake from R to P is 30 km, from R to Q is 33 km and $\angle RPQ$ is 60°.

(a) Show this information on the diagram below. (1 mark)



(b) If the distance from P to Q is d, use the cosine rule to show that $d^2 - 30d - 189 = 0$. (2 marks)

$$33^2 = 30^2 + d^2 - 2 \times 30 \times d \times \cos 60$$

$$1089 = 900 + d^2 - 30d$$

$$d^2 - 30d - 189 = 0$$

(c) Hence calculate the distance between the campsites at P and Q. (1 mark)

$d = 35.3$ km

(d) Determine an expression for θ, the size of $\angle PQR$, but do not calculate θ. (2 marks)

$$\frac{\sin\theta}{30} = \frac{\sin 60}{33}$$

$$\theta = \sin^{-1}\left(\frac{10\sin 60}{11}\right)$$

Question 13 **(8 marks)**

In the year 1990, in a marine park it was estimated that there were approximately 900 dolphins and that their population was increasing at 2% per year. At the same time, in the park there were approximately 1500 turtles and their population was decreasing at the rate of 5.5% per year.

(a) Write two separate equations, one for the number of dolphins t years after the year 1990 and one for the number of turtles t years after the year 1990. (4 marks)

$$d = 900\ (1.02)^t$$

Turtles: $y = 1500\ (0.945)^t$

(b) How many turtles are there in the marine park in 2005? (2 marks)

$$y = 1500\ (0.945)^{15} = 645\ turtles$$

(c) After how many years was the number of dolphins greater than the number of turtles? (2 marks)

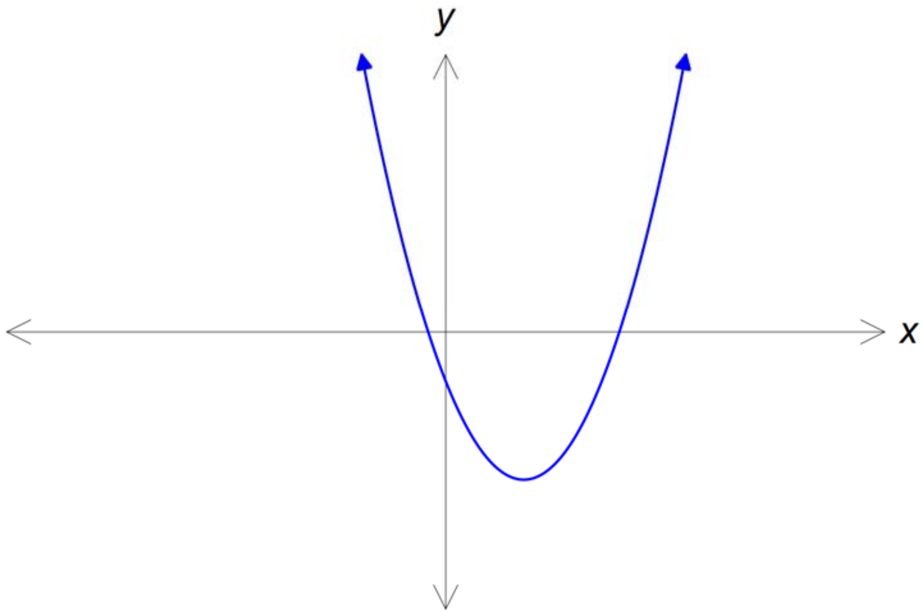
Solve:

$$900\ (1.02)^t = 1500\ (0.945)^t$$

$$t = 6.69\ years$$

Question 14 **(6 marks)**

Consider the function $g(x)$ below, with turning point (a, b) and y intercept $(0, c)$.



(a) Determine the coordinates of the turning point for $f(x)$, if $f(x) = g(x + 3) + 1$ (2 marks)

$$(a - 3, b + 1)$$

(b) Determine the coordinates of the y intercept for $p(x)$, if $p(x) = -g(x) + 5$ (2 marks)

$$(0, -c + 5)$$

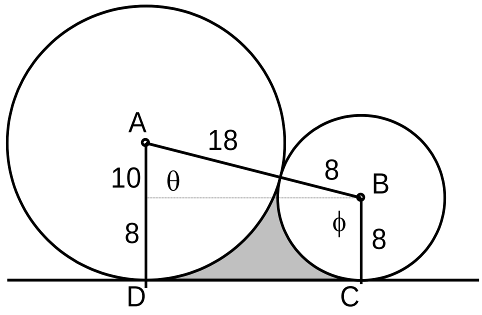
(c) State the domain and range of $m(x)$, if $m(x) = g(-x - 7)$. (2 marks)

Domain: $x \text{ is any real number}$

Range: $y \geq b$

Question 15 **(9 marks)**

Two circles, one of radius 8 cm and the other of radius 18 cm, with a common tangent, touch each other as shown in the diagram.



(a) Calculate the perimeter of the shaded region. (5 marks)

$CD = \sqrt{26^2 - 10^2} = 24$ cm

$\theta = \cos^{-1}\dfrac{10}{26} = 1.176^r$

$\phi = \pi - 1.176 = 1.966^r$

Long arc $= 18 \times 1.176 = 21.17$

Short arc $= 8 \times 1.966 = 15.72$

Perimeter $= 24 + 21.17 + 15.72 = 60.89$ cm

(b) Calculate the area of the shaded region. (4 marks)

Trapezium ABCD $= \dfrac{18+8}{2} \times 24 = 312$

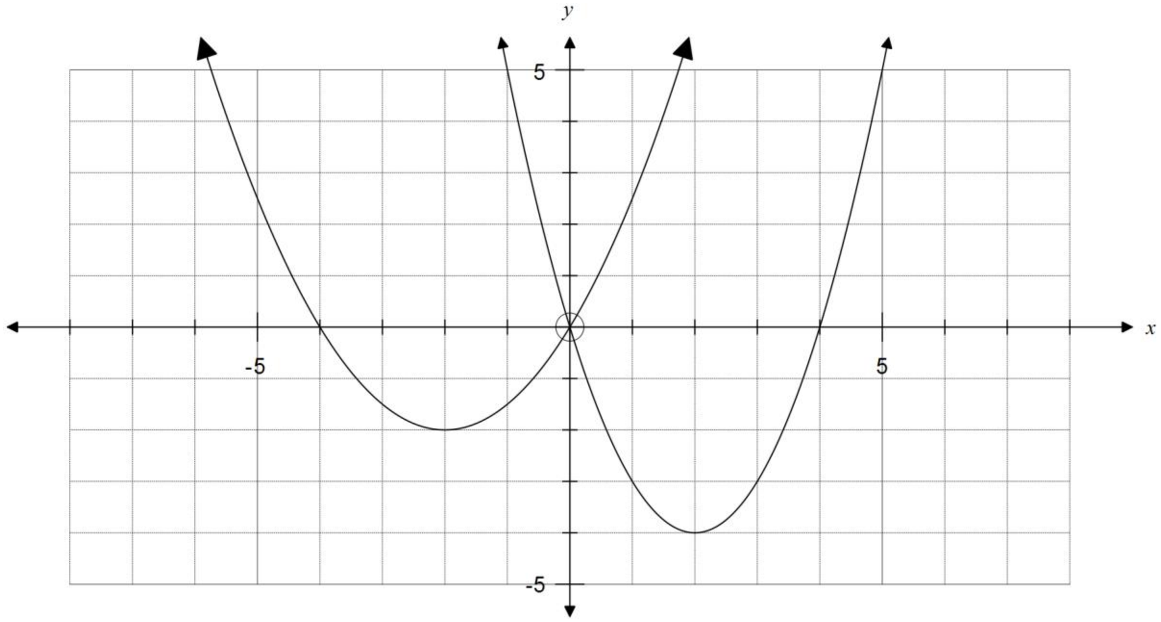
Large sector $= \dfrac{1}{2} \times 18^2 \times 1.176 = 190.51$

Small sector $= \dfrac{1}{2} \times 8^2 \times 1.966 = 62.90$

Total area $= 312 - 190.51 - 62.90 = 58.59$ cm^2

Question 16 **(8 marks)**

The function $y = f(x)$ is shown below.



(a) State the equation of $f(x)$ in the form $y = a(x+p)^2 + q$. (2 marks)

$$y = 0.5(x+2)^2 - 2$$

(b) State the domain and range of $f(x)$. (2 marks)

$$x \in \mathbb{R},\ y \geq -2$$

Another function is given by $g(x) = 2f(x-4)$.

(c) Describe the transformations required to produce $g(x)$ from $f(x)$. (2 marks)

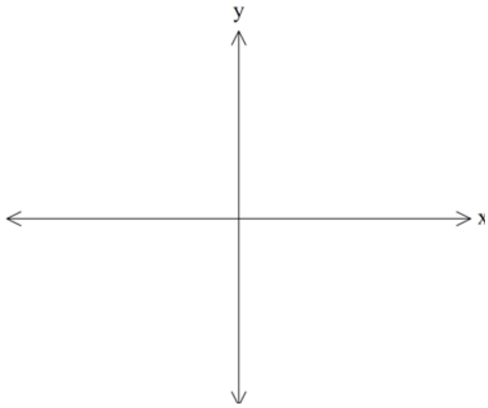
In either order:

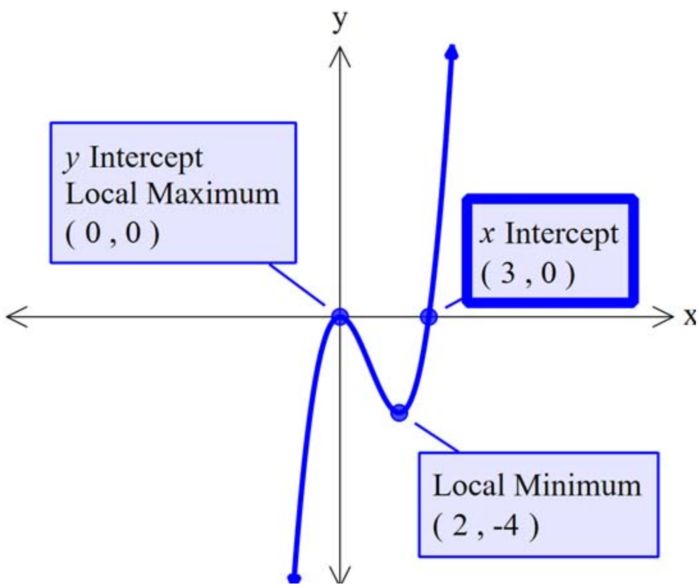
- translation of 4 units in the positive x direction
- vertical dilation of scale factor 2

(d) Draw the graph of $y = g(x)$ on the axes above. (2 marks)

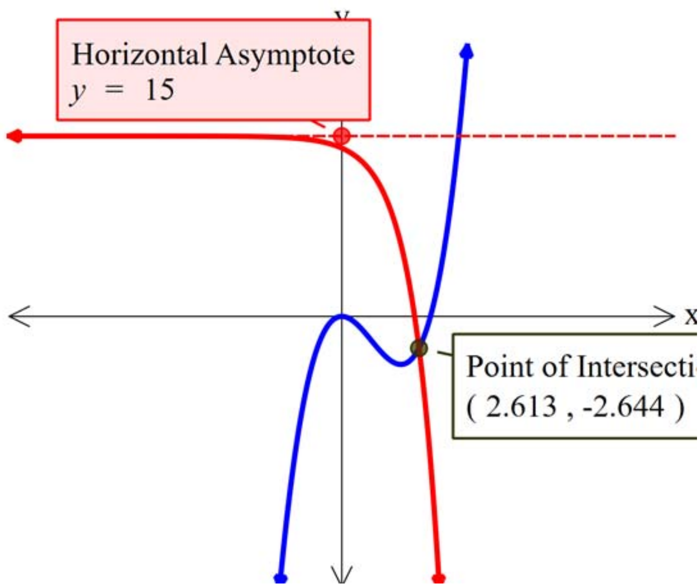
Question 17 **(11 marks)**

(a) Sketch the function $f(x) = x^3 - 3x^2$ on the axes below, labelling all intercepts and turning points. (4 marks)





(b) On the same axes, sketch the function $g(x) = 15 - 3^x$, labelling all key features. (4 marks)



(c) State the point of intersection of the graphs of $f(x)$ and $g(x)$ correct to 2 significant figures. (3 marks)

$(2.6, -2.6)$

Question 18 **(7 marks)**

(a) State the equation of the axis of symmetry for the graph $y = 2x^2 - 8x + 7$. (1 mark)

$$x = -\frac{-8}{2(2)}$$

$$x = 2$$

(b) Determine the discriminant of the quadratic equation $4x^2 - 20x + 25 = 0$ and hence state how many solutions the equation has. (2 marks)

$$(-20)^2 - 4(4)(25) = 400 - 400$$
$$= 0$$

Hence one solution.

(c) The parabola $y = ax^2 + bx - 10$ passes through the points (4.5, 8) and (-2.5, 15). Determine the values of a and b. (4 marks)

$$8 = 4.5^2 a + 4.5b - 10$$
$$15 = (-2.5)^2 a - 2.5b - 10$$

Solve simultaneously to get

$a = 2,\ b = -5$

Question 19 **(6 marks)**

The number of a certain species of crab, in Shark Bay, can be modelled by the function:

$$n(t) = 8\ (1.003)^t$$

Where t is the number of years and $n(t)$ is the number in thousands.

(a) What is the percentage increase in crabs each year? (1 mark)

$$0.3\%\ increase$$

(b) What is the crab population in 7 years? (1 mark)

$n(7) = 8.16951$ thousand (8170)

(c) After how many years will the number of crabs reach ten thousand? (2 marks)

$$solve:\ \ 10 = \ 8\ (1.003)^t$$

$$t = 74.5\ years$$

Another population of crabs can be modelled by:

$$m(t) = 3\ (1.04)^t$$

Where t is the number of years and $m(t)$ is measured in thousands.

(d) Determine when the second species of crabs will outnumber the first species. (2 marks)

Solve: $8\ (1.003)^t = 3\ (1.04)^t$

$t = 27.1\ years$

Question 20 **(5 marks)**

(a) Simplify $\sin(A+B)-\sin(A-B)$. (1 mark)

$2\cos(A)\cdot\sin(B)$

(b) Solve

(i) $2\sin^2 x-1=0$, $0\le x\le 2\pi$. (2 marks)

$\sin x=\pm\frac{1}{\sqrt{2}}$

$x=\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$

(ii) $2\cos^2 3x+\cos 3x=0$, $-90<x<90$. (2 marks)

$\cos 3x(2\cos 3x-1)=0$

$x=-80°,\ -40°,\ -30°,\ 30°,\ 40°,\ 80°$

Question 21 **(6 marks)**

(a) Use the angle sum and difference identities to show that

(i) $\cos(2A) = \cos^2 A - \sin^2 A$. (1 mark)

$$\begin{aligned} \cos 2A &= \cos(A + A) \\ &= \cos A\cos A - \sin A\sin A \\ &= \cos^2 A - \sin^2 A \end{aligned}$$

(ii) $\sin A = \cos(90^\circ - A)$. (1 mark)

$$\begin{aligned} \cos(90 - A) &= \cos 90\cos A + \sin 90\sin A \\ &= 0 \times \cos A + 1 \times \sin A \\ &= \sin A \end{aligned}$$

(b) The exact values of the sine and cosine of 36° are $\dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$ and $\dfrac{1+\sqrt{5}}{4}$ respectively.

Use both identities from (a) to show that the exact value of the sine of 18° is $\dfrac{\sqrt{5} - 1}{4}$.

(4 marks)

$$\begin{aligned} \sin 18 &= \cos(90 - 18) \\ &= \cos 72 \\ &= \cos(2 \times 36) \\ &= \cos^2 36 - \sin^2 36 \\ &= \left(\frac{1+\sqrt{5}}{4}\right)^2 - \frac{10 - 2\sqrt{5}}{16} \\ &= \frac{1 + 2\sqrt{5} + 5 - 10 + 2\sqrt{5}}{16} \\ &= \frac{\sqrt{5} - 1}{4} \end{aligned}$$

End of questions